# ÜSTEL FONKSİYON — LOGARİTMA
## KARMA SORU ÇÖZÜMÜ

(1) $9^{2x-5} = 27^{3x+1}$ ise $x=?$

$\Rightarrow (3^2)^{2x-5} = (3^3)^{3x+1}$

$\Rightarrow 3^{4x-10} = 3^{9x+3}$

$\Rightarrow 4x-10 = 9x+3$

$\Rightarrow -10-3 = 9x-4x$

$\Rightarrow \frac{-13}{5} = \frac{\cancel{5}x}{\cancel{5}} \Rightarrow \boxed{x = -\frac{13}{5}}$



(2) $(2x-4)^{17} = (3x+1)^{17}$ ise $x=?$

17 tek sayı : $2x-4 = 3x+1$

$\Rightarrow -4-1 = 3x-2x$

$\Rightarrow \boxed{-5 = x}$

(3) $(7x-2)^{120} = (2x+19)^{120}$ ise $x=?$

120 çift sayı : $7x-2 = 2x+19$

$\Rightarrow 5x = 21$

$\Rightarrow \boxed{x = \frac{21}{5}}$

$7x-2 = -(2x+19)$

$\Rightarrow 7x-2 = -2x-19$

$\Rightarrow 9x = -17$

$\Rightarrow \boxed{x = -\frac{17}{9}}$

4 $3^{4x-7} = 5^{6y-13}$ ise $x \cdot y = ?$

$4x-7=0 \qquad 6y-13=0$

$\Rightarrow 4x=7 \qquad \Rightarrow 6y=13$

$\Rightarrow \boxed{x=\frac{7}{4}} \qquad \Rightarrow \boxed{y=\frac{13}{6}}$

$x \cdot y = \frac{7}{4} \cdot \frac{13}{6} = \frac{91}{24}$

5 $\left.\begin{array}{l} 3^{2a-4} = 5^{2} \\ 9^{3a+1} = 125 \end{array}\right\}$ ise $a=?$

$\left.\begin{array}{l} 3^{2a-4} = 5^{2} \\ (3^{2})^{3a+1} = 5^{3} \end{array}\right\} \quad \left.\begin{array}{l} 3^{2a-4} = 5^{2} \\ 3^{6a+2} = 5^{3} \end{array}\right\} \quad \frac{2a-4}{6a+2} = \frac{2}{3}$

$6a-12 = 12a+4$

$\Rightarrow -12-4 = 12a-6a$

$\Rightarrow -16 = 6a \Rightarrow a = -\frac{16}{6}$

$\boxed{a = -\frac{8}{3}}$

Serdar Şakiroğlu
matematiksel.site

6) $(x-2)^{2x+6} = 1$ ise G.K. = ?

1) (Taban = 1) $x-2=1 \Rightarrow \boxed{x=3}$

2) $\begin{pmatrix} \text{Kuvvet} = 0 \\ \text{Taban} \neq 0 \end{pmatrix}$

$2x+6=0$

$2x=-6$

$\boxed{x=-3}$

Taban $= x-2$

$= -3-2$

$= -5 \neq 0$

3) $\begin{pmatrix} \text{Taban} = -1 \\ \text{Kuvvet çift} \end{pmatrix}$

$x-2=-1$

$\boxed{x=1}$

Kuvvet $= 2x+6$

$= 2 \cdot 1 + 6$

$= 8$ çift

G.K. $= \{-3, 1, 3\}$

$f(x) = a^x$ $\quad 0 < a < 1$ ise $f$ azalan

$a > 1$ ise $f$ artandır.

7) $f(x) = \left(\frac{3a-2}{5}\right)^x$ üstel fonksiyonu artan ise $a$ hangi aralıkta değerler alır?

$$\cancel{5} \cdot \frac{3a-2}{\cancel{5}} > 1 \cdot 5$$

$$\Rightarrow 3a - 2 > 5 \Rightarrow \frac{\cancel{3}a}{\cancel{3}} > \frac{7}{3} \Rightarrow \boxed{a > \frac{7}{3}}$$

$7/3$ $\quad \infty$

$$\boxed{G.A. = \left(\frac{7}{3}, \infty\right)}$$

8) $f(x) = \left(\frac{4a-7}{15}\right)^x$ üstel fonksiyonu azalan ise $a$'nın değer aralığını bulunuz.

Serdar Şakiroğlu
matematiksel.site

$$15 \cdot 0 < \frac{4a-7}{\cancel{15}} \cdot \cancel{15} < 1 \cdot 15$$

$$0 < 4a - 7 < 15$$

$$\frac{7}{4} < \frac{\cancel{4}a}{\cancel{4}} < \frac{22}{4} \Rightarrow \boxed{\frac{7}{4} < a < \frac{11}{2}}$$

$$G.A. = \left(\frac{7}{4}, \frac{11}{2}\right)$$

9 $f(x) = 5^{4x-3}$ ise $f^{-1}(x) = ?$

$x = 5^{4y-3}$

$\Rightarrow \log_5 x = 4y - 3$

$\Rightarrow 3 + \log_5 x = 4y \Rightarrow y = \dfrac{3 + \log_5 x}{4}$

$\Rightarrow \boxed{f^{-1}(x) = \dfrac{3 + \log_5 x}{4}}$

$\boxed{f^{-1}(x) = \frac{1}{4} \cdot (3 + \log_5 x)}$

Serdar Şakiroğlu
matematiksel.site

10 $f(x) = 4^{2x-7}$ ise $f^{-1}(x) = ?$

$x = 4^{2y-7}$

$\Rightarrow \log_4 x = 2y - 7$

$\Rightarrow 7 + \log_4 x = 2y \Rightarrow y = \dfrac{7 + \log_4 x}{2}$

$\boxed{f^{-1}(x) = \dfrac{7 + \log_4 x}{2}}$

$\boxed{f^{-1}(x) = \frac{1}{2} \cdot (7 + \log_4 x)}$

**11** $\left(\frac{3}{2}\right)^{2x+1} > \left(\frac{3}{2}\right)^{3x-2}$ ise $x$ hangi aralıkta değerler alır?

$\frac{3}{2}$ birden büyük. Eşitsizlik üslere aynen aktarılır.

$$2x+1 > 3x-2$$

$$\Rightarrow 2x-3x > -2-1$$

$$\Rightarrow \frac{\cancel{-}x}{\cancel{-}} > \frac{-3}{-1} \Rightarrow \boxed{x < 3}$$

$$\boxed{x \in (-\infty, 3)}$$

**12** $\left(\frac{2}{3}\right)^{5x-6} \leq \left(\frac{2}{3}\right)^{3x+1}$ ise $x \in$ ?

$\frac{2}{3}$ birden küçük: Eşitsizliği ters çevirip üslere aktarırız.

$$5x-6 \geq 3x+1$$

$$\Rightarrow 5x-3x \geq 1+6$$

$$\Rightarrow 2x \geq 7$$

$$\Rightarrow \boxed{x \geq \frac{7}{2}}$$


$$\boxed{x \in \left[\frac{7}{2}, \infty\right)}$$

13 $\left(\frac{4}{9}\right)^{2x-1} \leq \left(\frac{27}{8}\right)^{x+1}$ ise $x \in$ ?

$$\Rightarrow \left[\left(\frac{2}{3}\right)^{2}\right]^{2x-1} \leq \left[\left(\frac{2}{3}\right)^{-3}\right]^{x+1}$$

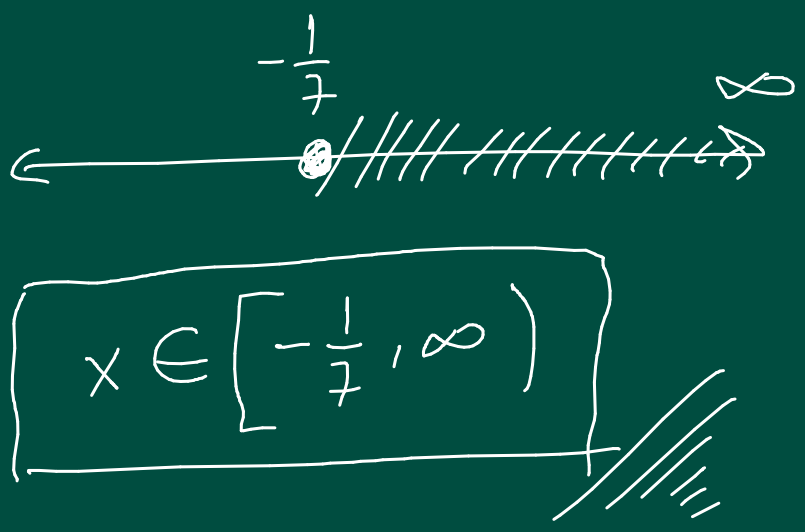
$$\Rightarrow \left(\frac{2}{3}\right)^{4x-2} \leq \left(\frac{2}{3}\right)^{-3x-3}$$

$\frac{2}{3}$ birden küçük: Eşitsizliğin yönünü çevirerek üslere aktaralım.

$$4x-2 \geq -3x-3$$

$$\Rightarrow 7x \geq -1$$

$$\Rightarrow \boxed{x \geq -\frac{1}{7}}$$




$$\boxed{x \in \left[-\frac{1}{7}, \infty\right)}$$

(14) $f(x) = \log_2(8x+1)$ ise $f^{-1}(x) = ?$

$x = \log_2(8y+1)$

$\log_2(8y+1) = x$

$\Rightarrow 8y+1 = 2^x$

$\Rightarrow 8y = 2^x - 1$

$\Rightarrow y = \dfrac{2^x-1}{8} \Rightarrow \boxed{f^{-1}(x) = \dfrac{2^x-1}{8}}$

$\Rightarrow \boxed{f^{-1}(x) = \dfrac{1}{8}\cdot(2^x-1)}$

(15) $f(x) = \log_5(2x+3)$ ise $f^{-1}(x) = ?$

$x = \log_5(2y+3)$

$\log_5(2y+3) = x$

$\Rightarrow 2y+3 = 5^x$

$\Rightarrow 2y = 5^x - 3$

$\Rightarrow y = \dfrac{5^x-3}{2} \Rightarrow \boxed{f^{-1}(x) = \dfrac{5^x-3}{2}}$

$\Rightarrow \boxed{f^{-1}(x) = \dfrac{1}{2}\cdot(5^x-3)}$

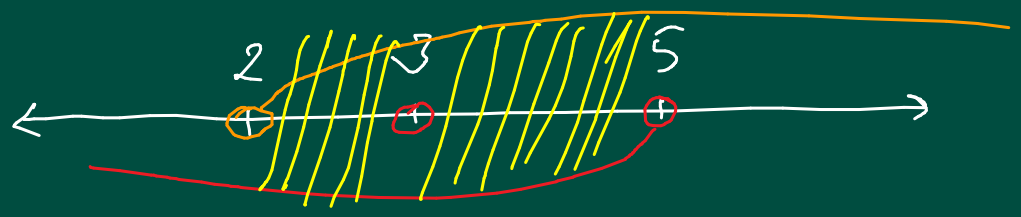
(16) $f(x) = \log_{(x-2)}(5-x)$ fonksiyonunun en geniş tanım kümesini bulunuz.

(1) Taban $>0$ $\quad x-2>0 \Rightarrow \boxed{x>2}$

(2) Taban $\neq 1$ $\quad x-2 \neq 1 \Rightarrow \boxed{x \neq 3}$

(3) Logaritmanın içindeki ifade $>0$

$$5-x>0 \Rightarrow 5>x \Rightarrow \boxed{x<5}$$




Tanım Kümesi $= (2,5) - \{3\}$

$= (2,3) \cup (3,5)$

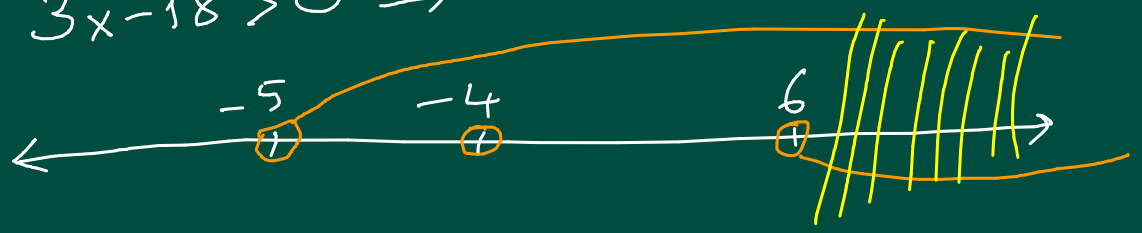
(17) $f(x) = \log_{(x+5)}(3x-18)$ fonksiyonunun en geniş tanım kümesi?



(1) $x+5>0 \Rightarrow \boxed{x>-5}$

(2) $x+5 \neq 1 \Rightarrow \boxed{x \neq -4}$

(3) $3x-18>0 \Rightarrow 3x>18 \Rightarrow \boxed{x>6}$




Tanım Kümesi $= (6, \infty)$

(18) $f(x) = \log_{(3-x)}(x^2-2x-15)$ fonksiyonunun en geniş tanım kümesi?

(1) $3-x>0 \Rightarrow 3>x \Rightarrow \boxed{x<3}$

(2) $3-x \neq 1 \Rightarrow 3-1 \neq x \Rightarrow \boxed{x \neq 2}$

(3) $x^2-2x-15>0$

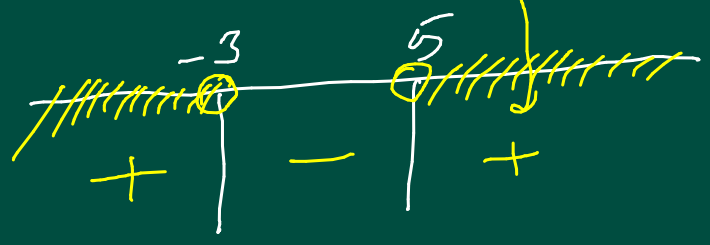
$x \longrightarrow -5$

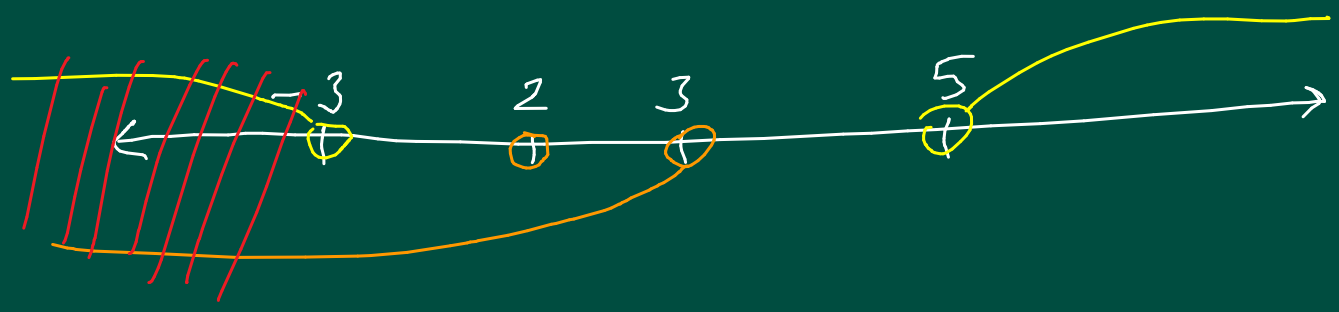
$x \longrightarrow +3$

$\underbrace{(x-5)}_{0} \cdot \underbrace{(x+3)}_{0} > 0$

$x-5=0$ $\quad$ $x+3=0$

$\boxed{x=5}$ $\quad$ $\boxed{x=-3}$




Tanım Kümesi $= (-\infty, -3)$

(19) $\cancel{\log}_3(2x-8)=4$ ise $x=?$

$\Rightarrow 2x-8=3^4$

$\Rightarrow 2x-8=81$

$\Rightarrow 2x=89 \Rightarrow \boxed{x=\frac{89}{2}}$

(20) $\cancel{\log}_2(\log_3(5x-1))=2$ ise $x=?$

$\Rightarrow \log_3(5x-1)=2^2$

$\Rightarrow \cancel{\log}_3(5x-1)=4$

$\Rightarrow 5x-1=3^4$

$\Rightarrow 5x-1=81$

$\Rightarrow 5x=82 \Rightarrow \boxed{x=\frac{82}{5}}$

21 $\cancel{\log_2}\left(2\cdot\log_3(x-2)\right)=1$ ise Ç.K = ?

$\Rightarrow 2\cdot\log_3(x-2)=2^1$

$\Rightarrow \dfrac{\cancel{2}\cdot\log_3(x-2)}{\cancel{2}}=\dfrac{2}{2}$

$\Rightarrow \cancel{\log_3}(x-2)=1$

Serdar Şakiroğlu
matematiksel.site

$\Rightarrow x-2=3^1$

$\Rightarrow x-2=3 \Rightarrow \boxed{x=5}$ Ç.K = $\{5\}$

22 $\cancel{\log_2}\left(1+2\cdot\log_3(x-4)\right)=1$ ise Ç.K. = ?

$\Rightarrow 1+2\cdot\log_3(x-4)=2^1$

$\Rightarrow 1+2\cdot\log_3(x-4)=2$

$\Rightarrow 2\cdot\log_3(x-4)=1$

$\Rightarrow \cancel{\log_3}(x-4)=\frac{1}{2} \Rightarrow x-4=3^{1/2}$

$\Rightarrow x-4=\sqrt{3}$

$\Rightarrow \boxed{x=4+\sqrt{3}}$

Ç.K. = $\{4+\sqrt{3}\}$

23

$\log_2(3x-12) < 1$ eşitsizliğinin çözüm aralığını bulunuz.

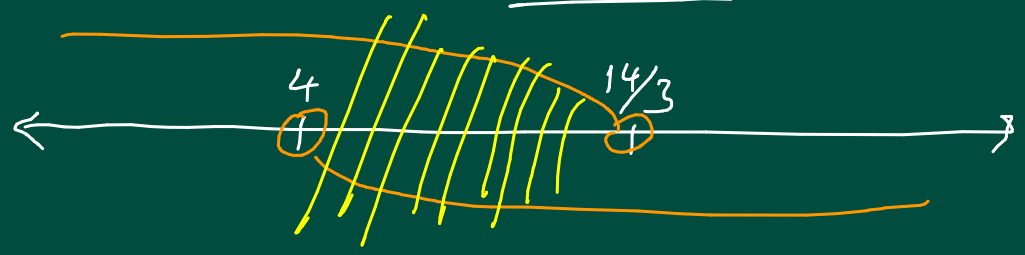
$\Rightarrow 3x-12 < 2^1$

$\Rightarrow 3x-12 < 2$

$\Rightarrow 3x < 14 \Rightarrow \boxed{x < \frac{14}{3}}$

Ayrıca logaritmalı ifadenin tanımlı olabilmesi için $3x-12>0$ olmalı.

$\Rightarrow 3x > 12 \Rightarrow \boxed{x > 4}$




$Ç = \left(4, \frac{14}{3}\right)$

24

$\log_{\frac{1}{3}}(2x-6) < 0$ eşitsizliğinin çözüm aralığını bulunuz.

$\Rightarrow 2x-6 > \left(\frac{1}{3}\right)^0 \rightarrow$ (Taban 1 den küçük olduğu için eşitsizlik yön değiştirdi)

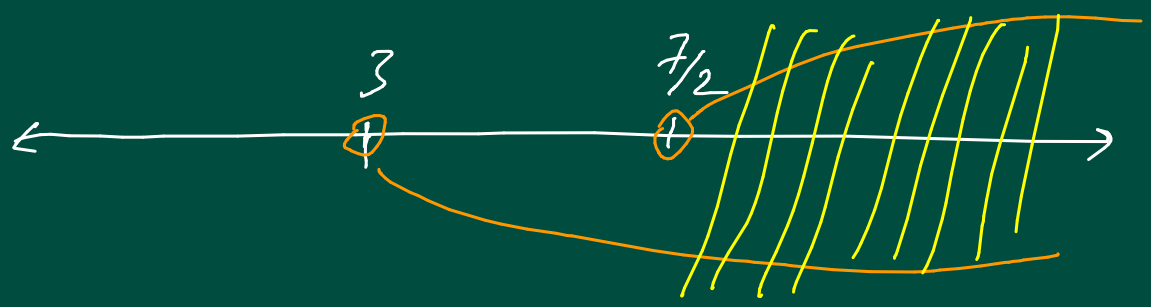
$\Rightarrow 2x-6 > 1$

$\Rightarrow 2x > 7 \Rightarrow \boxed{x > \frac{7}{2}}$

Ayrıca logaritmanın tanımlı olabilmesi için içindeki ifadenin sıfırdan büyük olması gerekir.

$2x-6 > 0$

$2x > 6 \Rightarrow \boxed{x > 3}$




$Ç = \left(\frac{7}{2}, \infty\right)$

**25** $1 \le \log_3(3x-6) \le 3$ eşitsizliğinin çözüm aralığı ?

$\Rightarrow 3^1 \le 3x-6 \le 3^3$

$\Rightarrow 3 \le 3x-6 \le 27$

$\Rightarrow \frac{9}{3} \le \frac{3x}{3} \le \frac{33}{3}$

$\Rightarrow 3 \le x \le 11 \Rightarrow Ç = [3, 11]$



**26** $\log_{\frac{1}{9}}(x+5) < \log_{\frac{1}{9}}(2x-7)$ $Ç = ?$

$\Rightarrow x+5 > 2x-7$

(Tabanlar 1 den küçük olduğu için eşitsizlik yön değiştirdi.)

$\Rightarrow x-2x > -7-5$

$\Rightarrow -x > -12$

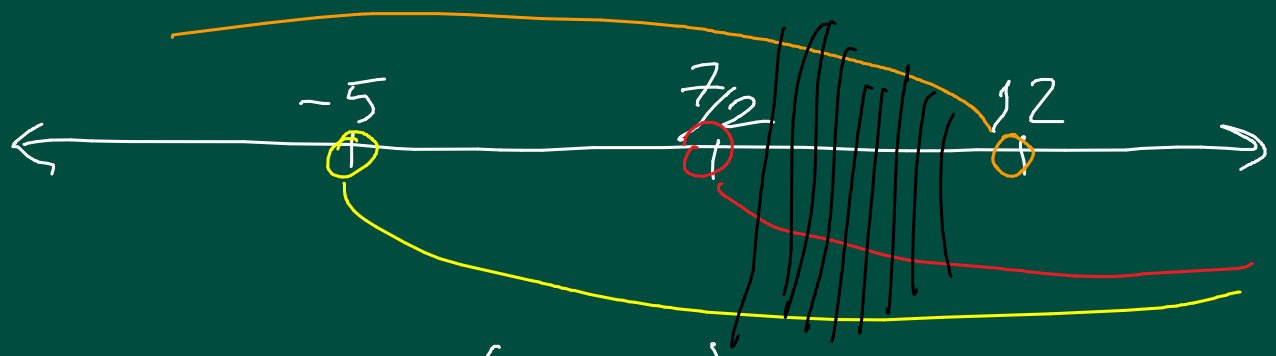
$\Rightarrow \boxed{x < 12}$

$x+5 > 0 \Rightarrow \boxed{x > -5}$

$2x-7 > 0$

$\Rightarrow 2x > 7$

$\Rightarrow \boxed{x > \frac{7}{2}}$




$Ç = \left(\frac{7}{2}, 12\right)$

**27**

$$\log_4 8 + \log 1000 - \ln e^5 = ?$$

$$= \log_{(2^2)}(2^3) + \log_{10}(10)^3 - \log_e e^5$$

$$= \frac{3}{2}\cdot\underbrace{\log_2 2}_{1} + 3\cdot\underbrace{\log_{10}10}_{1} - 5\cdot\underbrace{\log_e e}_{1}$$

$$= \frac{3}{2}\cdot 1 + 3\cdot 1 - 5\cdot 1$$

$$= \frac{3}{2} + 3 - 5 = \frac{3}{2} - 2 = -\frac{1}{2}$$

**28**

$$\log_{\left(\frac{1}{4}\right)} 32 - \log_{100} 10 + \ln e^2 - \underbrace{\log 10}_{1} = ?$$

$$= \log_{(2^{-2})}(2^5) - \log_{(10^2)}(10) + 2\cdot\underbrace{\ln e}_{1} - 1$$

$$= \frac{5}{-2}\cdot\underbrace{\log_2 2}_{1} - \frac{1}{2}\cdot\underbrace{\log_{10}10}_{1} + 2\cdot 1 - 1$$

$$= -\frac{5}{2}\cdot 1 - \frac{1}{2}\cdot 1 + 2 - 1$$

$$= -\frac{5}{2} - \frac{1}{2} + 1$$

$$= -\frac{6}{2} + 1 = -3 + 1 = -2$$

(29)

$$\cancel{3}^{\cancel{\log_3}(2x-4)} = x+1 \quad \text{ise} \quad x = ?$$

$$\Rightarrow 2x-4 = x+1$$

$$\Rightarrow \boxed{x=5}$$

(30)

$$\log_2 5 \cdot \log_{25} 9 \cdot \log_{27} 8 = ?$$

$$= \log_2 5 \cdot \log_{(5^2)} (3^2) \cdot \log_{(3^3)} (2^3)$$

$$= \log_2 5 \cdot \frac{2}{2} \cdot \log_5 3 \cdot \frac{3}{3} \cdot \log_3 2$$

$$= \cancel{\log_2 5 \cdot \log_5 3 \cdot \log_3 2}$$

$$= \log_2 2 = 1$$

Serdar Şakiroğlu

matematiksel.site

31

$$\frac{1}{\log_{25} 100} + \frac{1}{\log_{4} 100} = ?$$

$$= \log_{100} 25 + \log_{100} 4$$

$$= \log_{100}(25 \cdot 4)$$

$$= \log_{100} 100 = 1$$

32

$$\frac{1}{\log_{5} 80} + \frac{1}{\log_{2} 80} + \frac{1}{\log_{8} 80} = ?$$

$$= \log_{80} 5 + \log_{80} 2 + \log_{80} 8$$

$$= \log_{80}(5 \cdot 2 \cdot 8)$$

$$= \log_{80} 80 = 1$$

33

$\log_5\left(\frac{x}{y}\right)=3$ ve $\log_5(x\cdot y)=1$ ise $x=?$ $y=?$

$$\log_5 x - \cancel{\log_5 y}=3$$

$$+\quad \log_5 x + \cancel{\log_5 y}=1$$

$$2\cdot\log_5 x = 4$$

$$\Rightarrow \cancel{\log_5} x = 2 \Rightarrow x=5^2 \Rightarrow \boxed{x=25}$$

$$\log_5 x + \log_5 y = 1$$

$$\Rightarrow 2 + \log_5 y = 1$$

$$\Rightarrow \cancel{\log_5} y = -1$$

$$\Rightarrow y = 5^{-1} \Rightarrow \boxed{y=\frac{1}{5}}$$

Serdar Şakiroğlu

matematiksel.site